Find the general solution of the differential equation $y'' + 2(y')^2 \tan y = 0$. **SCORE: _____ / 8 PTS**

EACH ITEM ① POINT EXCEPT AS NOTED

Let $u = \frac{dy}{dx}$, so $\frac{d^2 y}{dx^2} = u\frac{du}{dy}$

② $u\frac{du}{dy} + 2u^2 \tan y = 0$

$\frac{du}{dy} + 2u \tan y = 0$ or $u = 0$

$\int \frac{1}{u}\, du = \int -2 \tan y\, dy$

$\frac{dy}{dx} = 0$ BONUS ①

$\ln|u| = 2\ln|\cos y| + C$

$y = K$

$u = C\cos^2 y$

$y'' + 2(y')^2 \tan y = 0 + 2(0)^2 \tan y = 0$ if $y \neq \frac{\pi}{2} + n\pi$

$\frac{dy}{dx} = C\cos^2 y$

So, $y = K \neq \frac{\pi}{2} + n\pi$ is also a solution

$\int \sec^2 y\, dy = \int C\, dx$

$\tan y = Cx + K$

$y = \arctan(Cx + K)$ NOTE: This solution includes the solution above by setting $C = 0$

Using elimination **as shown in lecture**, solve the system of differential equations **SCORE: _____ / 22 PTS**

$$5x' + 2y' + 6x - 3y = 9\cos 3t$$
$$2x' + y' + 2x - y = 0$$

$$(5D+6)[x] + (2D-3)[y] = 9\cos 3t$$
$$(2D+2)[x] + (D-1)[y] = 0$$

$$(D-1)(5D+6)[x] + (D-1)(2D-3)[y] = (D-1)[9\cos 3t] = -27\sin 3t - 9\cos 3t$$
$$(2D-3)(2D+2)[x] + (2D-3)(D-1)[y] = (2D-3)[0] = 0$$

$$((D-1)(5D+6) - (2D-3)(2D+2))[x] = -27\sin 3t - 9\cos 3t$$
(2) $$(D^2 + 3D)[x] = -27\sin 3t - 9\cos 3t$$
$$r = 0, -3$$
$$x_h = c_1 + c_2 e^{-3t}$$
$$x_p = A\sin 3t + B\cos 3t$$
$$x_p' = -3B\sin 3t + 3A\cos 3t$$
$$x_p'' = -9A\sin 3t - 9B\cos 3t$$
$$x_p'' + 3x_p' = (-9A - 9B)\sin 3t + (9A - 9B)\cos 3t$$
$$\begin{cases} -9A - 9B = -27 \\ 9A - 9B = -9 \end{cases} \Rightarrow \begin{cases} A + B = 3 \\ A - B = -1 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 2 \end{cases}$$
$$x = c_1 + c_2 e^{-3t} + \sin 3t + 2\cos 3t$$

$$(2D+2)(5D+6)[x] + (2D+2)(2D-3)[y] = (2D+2)[9\cos 3t] = -54\sin 3t + 18\cos 3t$$
$$(5D+6)(2D+2)[x] + (5D+6)(D-1)[y] = (5D+6)[0] = 0$$

$$((5D+6)(D-1) - (2D+2)(2D-3))[y] = 54\sin 3t - 18\cos 3t$$
$$(D^2 + 3D)[y] = 54\sin 3t - 18\cos 3t$$
$$y_h = k_1 + k_2 e^{-3t}$$
$$y_p = C\sin 3t + E\cos 3t$$
$$\begin{cases} -9C - 9E = 54 \\ 9C - 9E = -18 \end{cases} \Rightarrow \begin{cases} C + E = -6 \\ C - E = -2 \end{cases} \Rightarrow \begin{cases} C = -4 \\ E = -2 \end{cases}$$
$$y = k_1 + k_2 e^{-3t} - 4\sin 3t - 2\cos 3t$$

$$2x' + y' + 2x - y = \begin{array}{l} -6c_2 e^{-3t} - 12\sin 3t + 6\cos 3t \\ -3k_2 e^{-3t} + 6\sin 3t - 12\cos 3t \\ +2c_1 + 2c_2 e^{-3t} + 2\sin 3t + 4\cos 3t \\ -k_1 - k_2 e^{-3t} + 4\sin 3t + 2\cos 3t \end{array} = (2c_1 - k_1) + (-4c_2 - 4k_2)e^{-3t} = 0$$

(3)

$$\begin{cases} 2c_1 - k_1 = 0 \\ -4c_2 - 4k_2 = 0 \end{cases} \Rightarrow \begin{cases} k_1 = 2c_1 \\ k_2 = -c_2 \end{cases}$$

$$x = c_1 + c_2 e^{-3t} + \sin 3t + 2\cos 3t$$
$$y = 2c_1 - c_2 e^{-3t} - 4\sin 3t - 2\cos 3t$$